N87-18869

SENSITIVITY DERIVATIVES AND OPTIMIZATION OF NODAL POINT LOCATIONS FOR VIBRATION REDUCTION

Jocelyn I. Pritchard and Howard M. Adelman
NASA Langley Research Center
Hampton, Virginia

Raphael T. Haftka
Virginia Polytechnic Institute and
State University
Blacksburg, Virginia

ABSTRACT

A method is developed for sensitivity analysis and optimization of nodal point locations in connection with vibration reduction. A straightforward derivation of the expression for the derivative of nodal locations is given, and the role of the derivative in assessing design trends is demonstrated. An optimization process is developed which uses added lumped masses on the structure as design variables to move the node to a preselected location; for example, where low response amplitude is required or to a point which makes the mode shape nearly orthogonal to the force distribution, thereby minimizing the generalized force. The optimization formulation leads to values for added masses that adjust a nodal location while minimizing the total amount of added mass required to do so. As an example, the node of the second mode of a cantilever box beam is relocated to coincide with the centroid of a prescribed force distribution, thereby reducing the generalized force substantially without adding excessive mass. A comparison with an optimization formulation that directly minimizes the generalized force indicates that nodal placement gives essentially a minimum generalized force when the node is appropriately placed.

INTRODUCTION

The current trend in engineering design of aircraft and spacecraft is to incorporate in an integrated manner various design requirements and to do so at an early stage in the design process (refs. 1, 2). Incorporation of vibration design requirements is one example of this. Work in this area is ongoing in the Interdisciplinary Research Office at the Langley Research Center, particularly for vibration reduction in rotorcraft.

In helicopter rotor blade and fuselage design, stringent requirements on ride comfort, stability, fatigue life of structural components, and stable locations for electronic equipment and weapons lead to design constraints on vibration levels (refs. 3-5). Some of the methods previously used to control structural vibration in rotor blades include pendulum absorbers (ref. 6), active isolation devices (ref. 7), additional damping (refs. 5, 8), vibration absorbers which create "anti-resonances" (refs. 9, 10), and tuning masses to place frequencies away from driving frequencies (refs. 5, 11-14). Efforts to incorporate the above concepts for vibration reduction in systematic optimization techniques are described in references 10, 15-19. References 20, 21 contain surveys of applications of optimization methods for vibration control of helicopters.

The objectives of this paper are to develop and demonstrate the concept of nodal point placement and develop a mathematical optimization procedure based on this concept to reduce vibration. An important ingredient in the optimization procedure is the derivative of the nodal point location with respect to a design variable. This derivative quantifies the sensitivity of a nodal location to a change in a design variable. The sensitivity derivative of the nodal location is derived in this paper. The equation involves the derivative of the vibration mode with respect to the design variable and the slope of the mode shape at the nodal point and is easily implemented in a vibration analysis program using available or easily computed quantities. Analytical results are presented for the sensitivity derivatives for a beam model of a rotor blade and compared with finite differences for an independent check. The sensitivity derivatives have been employed in an optimization procedure for placing a node at a specified location by varying the sizes of lumped masses while minimizing the sum of these masses. Optimization results are shown for placement of a node at a prescribed location on the beam model.

Recently, the concept of "modal shaping" has been proposed as a method to reduce structural vibration, especially in helicopters (refs. 3, 4). In this method, vibration modes of rotor blades are altered through structural modification to make them nearly orthogonal to the air load distribution - thus reducing the generalized (modal) force. This paper deals with the concept of nodal point placement which is related to modal shaping and consists of modifying the mass distribution of a structure to place the node of a mode at a desirable location. Typical candidates for nodal point placement are locations where low response amplitude is required such as pilot or passenger seats, locations of sensitive electronic equipment, weapon platforms, or engine mounts. Nodal point placement also has the potential for reducing overall response by placing a node at a strategic location of a force distribution to reduce the generalized force.

MOTIVATION FOR DERIVATIVES OF NODAL POINT LOCATIONS

A method has been developed for calculating the sensitivity derivatives of node locations (points of zero displacement on a mode shape). These derivatives are used in optimization procedures to place nodes for the purpose of reducing vibrations. There are two general cases of nodal placement (figure 1). The first case places a node at a point where low response is desirable such as the pilot or passenger seat, the location of sensitive electronics, or weapon platforms, for example. The second case places the node at a point to minimize the generalized force. By placing the node at certain locations, the major components of the force vector are cancelled out and, therefore, the generalized force is reduced. Two possible candidates for placement of the node in this case are the point of maximum force or the centroid of the force. An example of the latter will be shown. The derivatives of nodal locations, besides being used in optimization procedures to place nodes, provide valuable information about the effect of a design change in moving the location of the node.

- Application of nodal placement

  Points desirable for low response

  - Pilot or passenger seat
  - Location of sensitive electronics
  - Weapon platform

  Minimize generalized force, $\Phi^T F$

- Design application

  Tells which design variables are most effective in changing nodal location

Figure 1

DERIVATIVES OF NODAL POINT LOCATIONS

The derivation of the analytical expression used to calculate the derivatives of node point locations is developed for an arbitrary design variable, v. The modal deflection normal to the length of a one-dimensional structure is denoted u(x,v) and represented by the solid line in the sketch of figure 2. The deflection, u and the nodal point location denoted by $x_n(v)$ are both functions of a design variable. When the design variable is perturbed, the deflection shape changes to the shape shown by the dashed line. The derivative of the nodal location is obtained by expanding the perturbed mode in a Taylor series about the nominal nodal point. Neglecting the higher order terms,

$$u(x_n + dx_n, v + dv) = u(x_n, v) + \left.\frac{\partial u}{\partial x}\right|_{x_n, v} dx_n + \left.\frac{\partial u}{\partial v}\right|_{x_n, v} dv \qquad (1)$$

The term on the left side of the equation and the first term on the right are deflections at the nodal points of the perturbed and nominal mode shapes, respectively, which are zero. Since x_n is a function of v, it follows that $dx_n = \frac{dx_n}{dv} dv$. Therefore, from (1)

$$\left.\frac{\partial u}{\partial x}\right|_{x_n, v} dx_n + \left.\frac{\partial u}{\partial v}\right|_{x_n, v} dv = \left(\left.\frac{\partial u}{\partial x}\right|_{x_n, v} \frac{dx_n}{dv} + \left.\frac{\partial u}{\partial v}\right|_{x_n, v}\right) dv = 0 \qquad (2)$$

Noting that dv is arbitrary and solving for dx_n/dv leads to the formula for the nodal point derivative

$$\frac{dx_n}{dv} = - \left[\frac{\partial u/\partial v}{\partial u/\partial x} \right]_{x_n, v} \qquad (3)$$

The two ingredients in the formula are $\partial u/\partial v$, the derivative of the mode shape at the nodal point and $\partial u/\partial x$, the slope of the mode shape at the nodal point. The value of $\partial u/\partial x$ is obtained from the nominal mode shape; and the value of $\partial u/\partial v$ is obtained by Nelson's method (ref. 22) which will be described in the next figure.

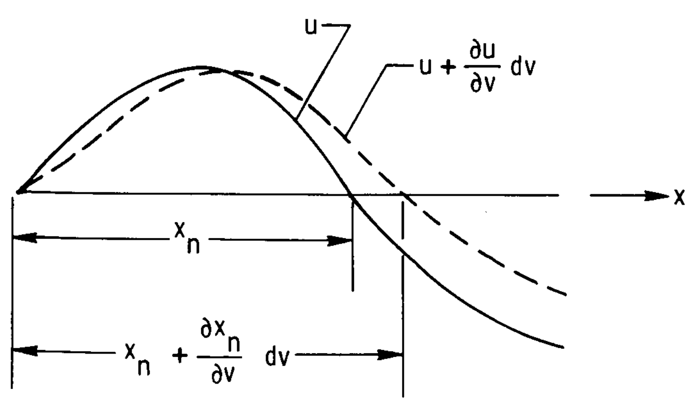


Figure 2

DERIVATIVES OF EIGENVECTORS - NELSON'S METHOD*

A free-vibration problem with no damping is governed by equation (4) of figure 3 where K is the stiffness matrix, M is the mass matrix, ϕ is the eigenvector, and λ is the eigenvalue (square of the circular frequency). The eigenvector is normalized such that the generalized mass is unity (eq. (5)). By taking the derivative of equation (4) with respect to a design variable v, equation (6) is obtained. Because this equation is singular, a direct solution for $\frac{\partial\phi}{\partial v}$ is not possible. However, the general solution to equation (6) is expressible in the form of equation (7) as the sum of a complementary solution, ϕ and a particular solution, Q. The particular solution is found by setting one component of the eigenvector derivative equal to zero and deleting the corresponding row and column from equation (6) and solving for the remaining components. The constant C is found by taking the derivative of the normalization condition in equation (5) and substituting equation (7) into the resulting expression.

- $\{K - \lambda M\}\Phi = 0$ (4)

- $\Phi^T M \Phi = 1$ (5)

- Take derivative of Eq. (4)

$$\{K - \lambda M\}\frac{\partial\Phi}{\partial v} = \frac{\partial\lambda}{\partial v} M\Phi - \frac{\partial K}{\partial v}\Phi + \lambda\frac{\partial M}{\partial v}\Phi \qquad (6)$$

- Solution: $\frac{\partial\Phi}{\partial v} = Q + C\Phi$ (7)

- C is determined from derivative of Eq. (5)

$$2\Phi^T M\frac{\partial\Phi}{\partial v} = -\Phi^T\frac{\partial M}{\partial v}\Phi$$

- $C = -\Phi^T MQ - \frac{1}{2}\Phi^T\frac{\partial M}{\partial v}\Phi$

*Ref. 22

Figure 3

For calculating derivatives of nodal locations of spinning structures such as rotor blades, a modification of the previous development is necessary. The basic expression for the nodal point derivative is unaffected (see eq. (3)), and Nelson's method is still used to calculate the eigenvector derivative. However, the details of Nelson's method when applied to a spinning structure are different because the eigenvalue problem has additional stiffness terms (refs. 23, 24). As shown in figure 4, the new terms are K_C, the centrifugal stiffness matrix and K_D, the differential stiffness matrix. K_C contains products of masses m and angular velocity Ω. K_D contains stresses associated with the extension of the spinning structure. (Details may be found in refs. 23 and 24.) Presently, the derivative of the stiffness matrix is calculated by finite differences, but methods for calculating this derivative analytically are being investigated.

- Stiffness matrix for spinning structures

  $$K = K_E + K_C + K_D$$

  - K_E = elastic stiffness matrix
  - K_C = centrifugal stiffness matrix = (m, Ω)
  - K_D = differential stiffness matrix = $K_D(\sigma)$

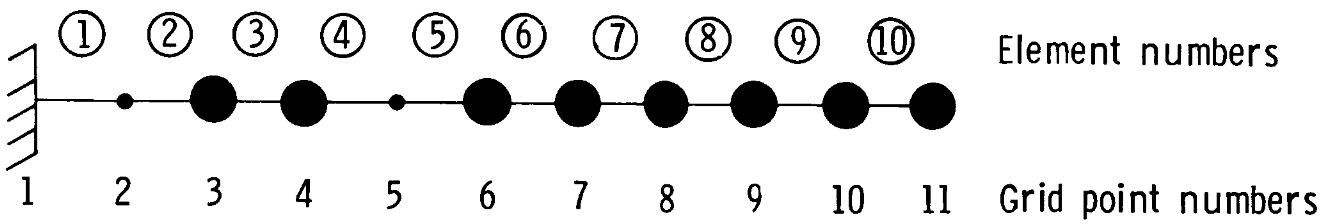
- Compute $\partial(K_E + K_C + K_D)/\partial v$ by finite differences

- Plans are to develop derivative of $K_E + K_C + K_D$ analytically

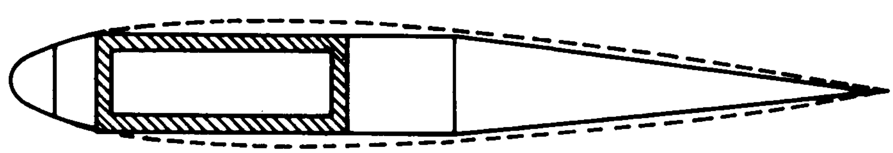
Figure 4

SENSITIVITY DERIVATIVE TEST PROBLEM

The example problem used to test the sensitivity analysis is a cantilever beam representation of a rotor blade developed in reference 25 and shown in figure 5. The beam is 193 inches long and is modeled by ten finite elements of equal length. The model contains both structural mass and lumped (non-structural) masses. The beam has a box cross section as shown in the figure. Additional details of the model are given in reference 26. There are eight lumped masses at various locations along the length of the beam and the values of the masses are the design variables. The derivatives of the nodal location with respect to these lumped masses are computed for the second mode. The second mode is chosen because it is a prime contributor to the vibrations transmitted from the rotor to the fuselage (ref. 3).



Finite element model



Cross-sectional detail

- Compute derivatives of node location for second mode
- Design variables - masses at grid points

Figure 5

RESULTS OF SENSITIVITY ANALYSIS

Derivatives of the nodal point location with respect to the lumped masses for the second mode were calculated using equation (3). The sensitivity analysis included the model with spin (Ω=425 rpm), as well as without (Ω=0). For an independent check on the implementation of equation (3), the derivatives were also calculated by forward finite differences with a step size of .1 percent. The sensitivity results are shown in figure 6. The two methods generally agreed within one percent. Examination of the table shows both positive and negative values of the derivatives. A positive value indicates that an increase in the mass moves the nodal point to the right of the nominal location and a negative value indicates that an increase in mass moves the node to the left. The derivatives show, for example, that changes in the masses at grid points 10 and 11 are the most effective ways (per unit mass) to move the node. The derivatives for the spinning model follow the same basic trends as the non-spinning model even though the derivatives are somewhat different.

dx_n/dv (inch/lbm)

	$\Omega = 0$		$\Omega = 425$ rpm	
Mass no.	Analytical	Finite difference	Analytical	Finite difference
3	-0.028	-0.028	-.050	-.050
4	-0.088	-0.088	-.129	-.129
6	-0.231	-0.230	-.261	-.261
7	-0.236	-0.236	-.221	-.221
8	-0.166	-0.165	-.096	-.096
9	-0.004	-0.004	.062	.062
10	0.309	0.309	.280	.280
11	0.828	0.826	.778	.777

Figure 6

OPTIMIZATION TO PLACE NODES

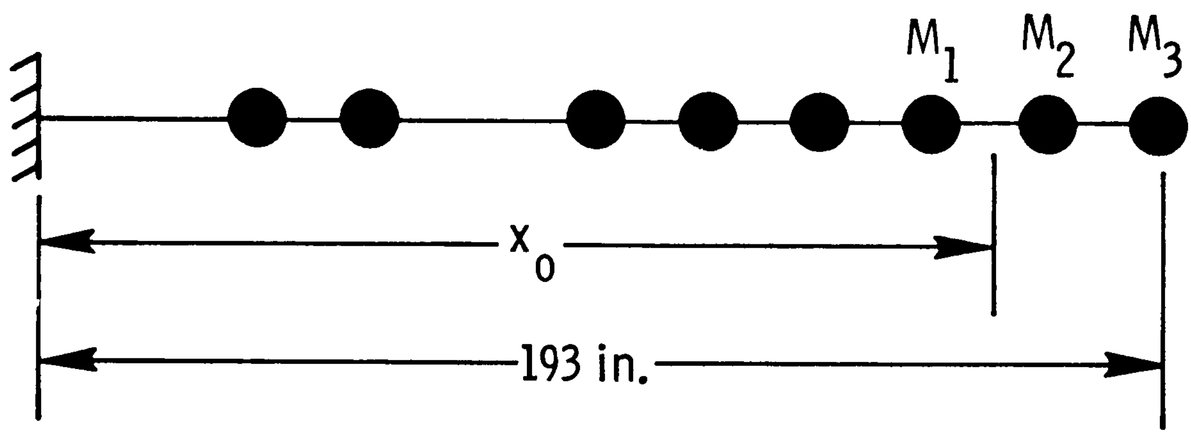
The optimization problem is to place the node at a desired location by varying the magnitudes of lumped masses while minimizing the total lumped mass. CONMIN, a general-purpose optimization program (ref. 27), is utilized. The formulation of the problem consists of defining an objective function (the quantity to be minimized); the constraints (limitations on the behavior of the model); and the design variables (the parameters of the model to be changed in order to find the optimum design). The optimizer requires derivatives of both the objective function and the constraints. The formulation for this problem, summarized in figure 7, is as follows: The objective function, f, is the sum of the lumped masses. The constraint, g, which must be negative or zero for an acceptable design, expresses the requirement that the nodal point x_n be placed within a distance δ from a desired location x_o. The design variables consist of the sizes of the lumped masses. Constraints on the largest and smallest acceptable values of the design variables are optional. These values are arbitrarily set in this case. The derivatives of the objective function with respect to the design variables are all equal to 1.0 and the derivatives of the constraints are equal to positive or negative values of the nodal point sensitivity derivatives calculated from equation (3).

- Problem: Place node x_n within δ of x_0 by varying masses M_i
- Objective function, $f = \sum_{i=1} M_i$
- Constraint, $g = |x_n - x_0| - \delta \leq 0$
- Design variables, $v_i = M_i$
- Use CONMIN
- Derivatives of objective function: $\partial f / \partial v_i = 1.0$
- Derivatives of constraints: $\partial g / \partial v_i = \pm \partial x_n / \partial v_i$

Figure 7

OPTIMIZATION TEST PROBLEM

The model used in the optimization procedure is shown in figure 8 and is the same beam structure of figure 5. The node for the second mode is to be placed within $\delta = 1.0$ inch of $x_o = 164$ inches. The location x_o is chosen because it is the centroid of a representative air load distribution given in reference 3 for a rotor blade. In reference 26, it is shown that the centroid of a load distribution is a desirable location for the node. The design variables are the masses at joints 9, 10, and 11 having initial values of 5.21 lbm, 6.55 lbm, and 6.60 lbm, as given in reference 25 - a total of 18.36 pounds. The initial location of the node is 154.7 inches. The upper and lower bounds on the design variables are 50.0 and 0.5 lbm, respectively.



- Desired node location : $x_0 = 164.0$ in.
- Allowable distance: $\delta = 1.0$ in.
- Design variables: M_1 M_2 M_3
- Upper bounds on design variables: 50 lb
- Lower bounds on design variables: 0.5 lb

Figure 8

CONVERGENCE OF OPTIMIZATION PROCEDURE FOR NODAL LOCATION

Initially, the constraint is not satisfied since the node is nine inches from the desired location (instead of one inch). The optimization history is shown in figure 9. The optimizer initially adds mass to bring the nodal point to within one inch of the desired location. After ten cycles, the constraint is satisfied, but the mass is increased to about 36 lbm. For the remainder of the cycles, the optimizer concentrates on minimizing the total mass by shifting mass among the three locations, finally reaching the optimum design with a mass of 24.45 lbm.

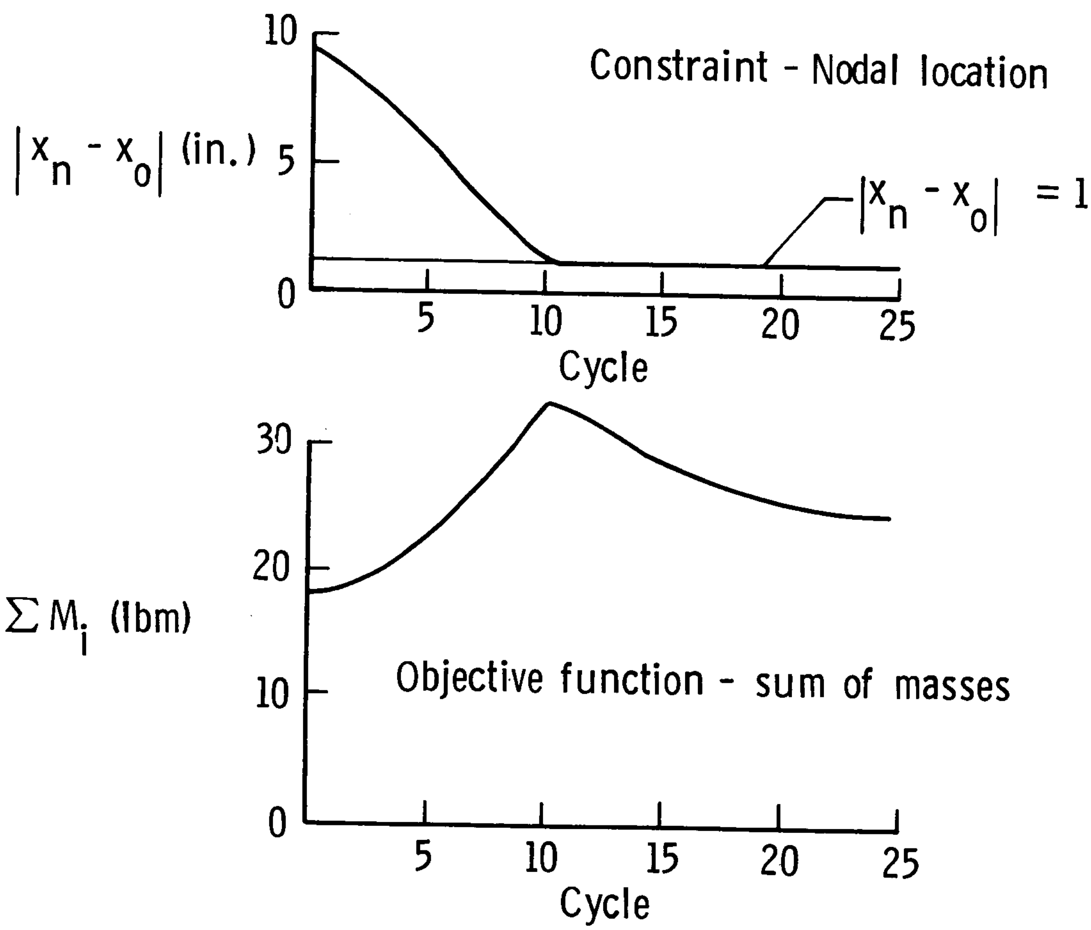


Figure 9

INITIAL AND FINAL DESIGN FOR NODAL POINT OPTIMIZATION

The optimization procedure converged to the final design shown in figure 10 in which the masses are 0.5 lbm, 3.70 lbm, and 20.25 lbm, for a total of 24.45 lbm, and the nodal point is located at 163 inches. Basically, mass was shifted from the two inboard locations to the tip where mass is most effective in moving the nodal point. For example, the mass at grid point 9 is reduced from 5.21 lbs to 0.5 lbs, while the tip mass is increased from 6.6 lbs to 20.25 lbs. Excessive addition of mass is avoided (only 6 additional pounds were needed) because of the effectiveness of relocating mass to the tip.

$x_0 = 164.0$ in. $\delta = 1.0$ in.

	Initial	Final
M_1 (lbm)	5.21	0.50
M_2 (lbm)	6.55	3.70
M_3 (lbm)	6.60	20.25
M_{TOT} (lbm)	18.36	24.45
Nodal location x_n (in.)	154.7	163.0

Figure 10

GENERALIZED FORCE STUDY

One of the potential applications of nodal point placement is the reduction of overall vibration response by generalized force minimization. A study is performed in which the generalized force for the second mode is calculated using the force distribution F, shown in figure 11. This generalized force is $\phi_2^T F$ where ϕ_2 is the mode shape from the final design based on the nodal point placement optimization. The force distribution in figure 11 is taken from reference 3 as representative of the air loading on a rotor blade and is adjusted so that the centroid is near the location of the nodal point; i.e., (164 ± 1) inches. Locating the node at the centroid results in a low value for the generalized force (ref. 26). To assess how well nodal placement reduces generalized force, the generalized force from node placement optimization is compared with the value obtained when the generalized force is directly minimized (ref. 26).

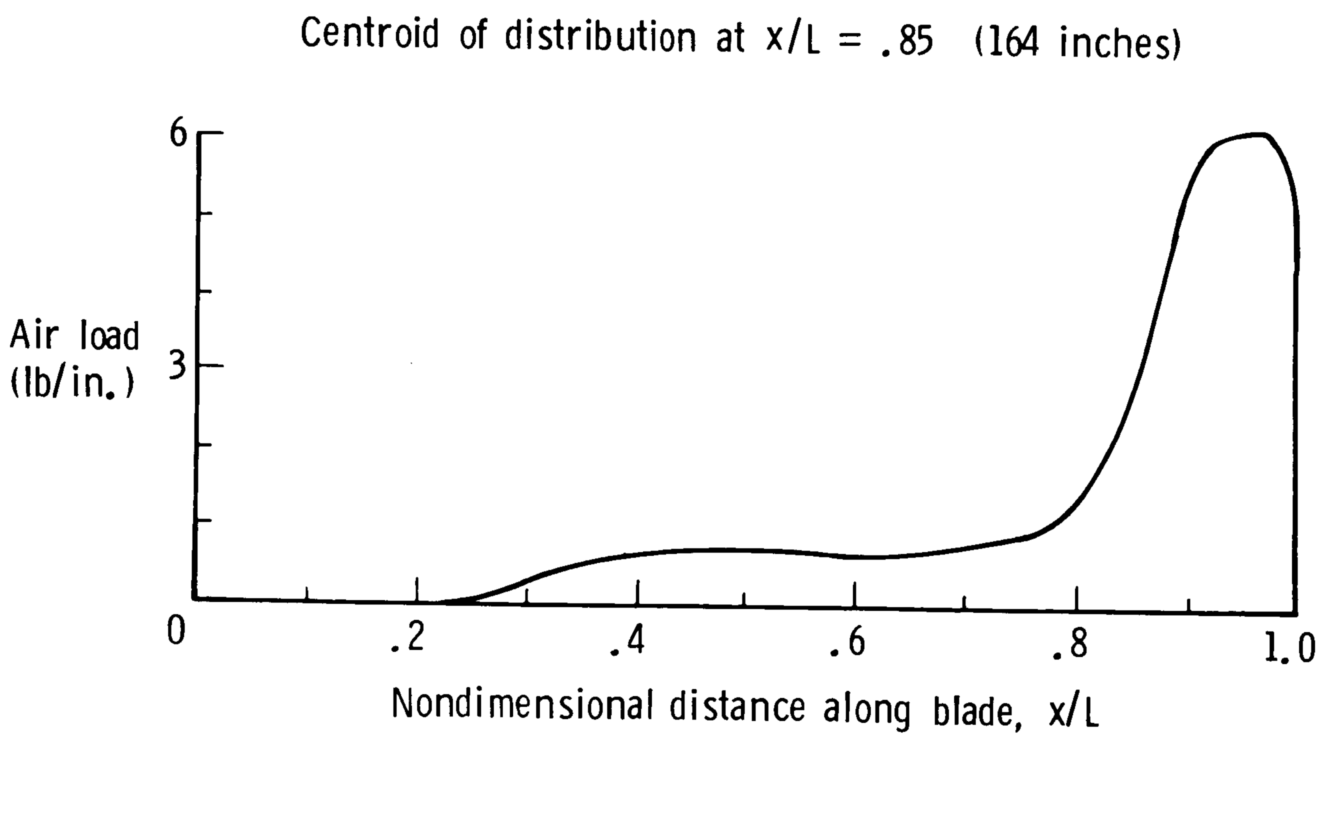


Figure 11

DESIGN CHARACTERISTICS FROM NODAL POINT OPTIMIZATION

Figure 12 contains design variables, total mass, generalized force, and nodal point locations for three designs: the initial design, the final design from nodal placement, and the final design from the direct minimization of the generalized force (ref. 26). The nodal placement procedure is very effective in minimizing the generalized force - giving 10.8 lbf, compared to 10.0 lbf from the direct method when both were started at a design with a generalized force of 20.8 lbf. The direct minimization procedure, while not dealing directly with the nodal location, nevertheless places the node essentially at the same point as the nodal placement design: 163.8 inches versus 163.0 inches.

	Initial	Nodal placement	Direct minimization
Generalized force (lbf)	20.8	10.8	10.0
Nodal location x_n (inch)	154.70	163.0	163.8
M_1 (lbm)	5.21	0.50	0.50
M_2 (lbm)	6.55	3.70	1.75
M_3 (lbm)	6.60	20.25	22.20
M_{TOT} (lbm)	18.36	24.45	24.45

Figure 12

CONCLUDING REMARKS

This paper has described sensitivity analysis and optimization methods for adjusting mode shape nodal point locations with application to vibration reduction. The paper begins with a derivation of an expression for the derivative of the nodal location with respect to a design variable. Sensitivity analyses were performed on a demonstration problem which consisted of a box beam model of a helicopter rotor blade. In these analyses, the derivatives of the nodal location for the second mode with respect to the magnitudes of lumped masses on the beam were calculated. It was shown that these derivatives gave useful information about the effect of the masses on the nodal location and indicated which masses were most effective in moving the nodal point. Next, the paper described an optimization procedure to place a node at a prescribed location by adjusting the magnitudes of lumped masses while minimizing the sum of these masses. A general-purpose optimization program was used and the nodal point derivatives were a key ingredient in the procedure. This optimization procedure was demonstrated in an example where the nodal point for the second mode of a cantilever beam model of a rotor blade was placed at a location close to the centroid of a force distribution. The procedure was successful in moving the node to the desired location requiring only six additional pounds of lumped mass on a 193-inch beam that weighed 117 pounds.

Finally, to demonstrate the potential for nodal placement to reduce vibration, the generalized force for the second mode was calculated and compared to the minimum generalized force obtained by a separate optimization procedure. It was found that the nodal placement procedure gave a generalized force which was very close to the minimum. The results in this paper suggest that adjusting the mode shapes of structures by relocating nodal points has potential for reducing both overall and local response levels in vibrating structures.

REFERENCES

1. Sobieszczanski-Sobieski, J.: Structural Optimization: Challenges and Opportunities. Paper presented at International Conference on Modern Vehicle Design Analysis,London, England, June 1983.

2. Sobieszczanski-Sobieski, J.; James B. B.; and Riley, M. F.: Structural Optimization by Generalized, Multilevel Optimization. AIAA Paper No. 85-0697-CP.

3. Taylor, R. B.: Helicopter Vibration Reduction by Rotor Blade Modal Shaping. Presented at the 38th Annual Forum of the American Helicopter Society, Anaheim, CA, May 1982.

4. Taylor, R. B.: Helicopter Rotor Blade Design for Minimum Vibration. NASA CR-3825, 1984.

5. Blackwell, R. H.: Blade Design for Reduced Helicopter Vibration. Journal of the American Helicopter Society, Vol. 28, No. 3, July 1983.

6. Hamouda, M. H.; and Pierce, G. A.: Helicopter Vibration Suppression Using Simple Pendulum Absorbers on the Rotor Blade. Presented at the American Helicopter Society Northeast Region National Specialists' Meeting on Helicopter Vibration, Hartford, CT, November 1981.

7. Reichert, G.: Helicopter Vibration Control - A Survey. Vertica, Vol. 5, 1981, pp. 1-20.

8. Rogers, L.: Damping as a Design Parameter. Mechanical Engineering, Vol. 108, No. 1, January 1986.

9. Wang, B. P.; Kitis, L.; Pilkey, W. D.; and Palazzolo, A.: Synthesis of Dynamic Vibration Absorbers. Journal of Vibration, Acoustics, Stress, and Reliability in Design, Vol. 107, April 1985, pp. 161-166.

10. Kitis, L.; Pilkey, W. D.; and Wang, B. P.: Optimal Frequency Response Shaping by Appendant Structures. Journal of Sound and Vibration, Vol. 95, No. 2, 1984, pp. 161-175.

11. Davis, M. W.: Optimization of Helicopter Rotor Blade Design for Minimum Vibration. NASA CP-2327, Part 2, September 1984, pp. 609-625.

12. McCarty, J. L.; and Brooks, G. W.: A Dynamic Model Study of the Effect of Added Weights and Other Structural Variations on the Blade Bending Strains of an Experimental Two-Blade Jet-Driven Helicopter in Hovering and Forward Flight. NACA TN--3367, May 1955.

13. Hirsh, H.; Hutton, R. E.; and Rasamoff, A.: Effect of Spanwise and Chordwise Mass Distribution on Rotor Blade Cyclic Stresses. Journal of the American Helicopter Society, Vol. 1, No. 2, April 1965, pp. 37-45.

14. Peers, D. A.; Rossow, M. P.; Korn, A.; and Ko, T.: Design of Helicopter Rotor Blades for Optimum Dynamic Characteristics. Comp. and Maths. with Appls., Vol. 12A, No. 1, 1986, pp. 85-109.

15. Friedmann, P. P.: Application of Modern Structural Optimization to Vibration Reduction in Rotorcraft. NASA CP-2327, Part 2, 1984, pp. 553-566.

16. Bennett, R. L.: Application of Optimization Methods to Rotor Design Problems. Vertica, Vol. 7, Part 3, 1983, pp. 201-208.

17. Friedmann, P. P.; and Shanthakumaran, P.: Aeroelastic Tailoring of Rotor Blades for Vibration Reduction in Forward Flight. Presented at 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, NV, May 2-4, 1983.

18. Friedmann, P. P.; and Shanthakumaran, P.: Optimum Design of Rotor Blades for Vibration Reduction in Forward Flight. Journal of the American Helicopter Society, October 1984.

19. Sutton, L. R.; and Bennett, R. L.: Aeroelastic/Aerodynamic Optimization of High-Speed Helicopter/Compound Rotor. NASA CP-2327, Part 2, September 1984, pp. 643-662.

20. Miura, H.: Applications of Numerical Optimization Methods to Helicopter Design Problems: A Survey. NASA TM-86010, October 1984.

21. Friedmann, P.: Application of Modern Structural Optimization to Vibration Reduction in Rotorcraft. Vertica, Vol. 9, No. 4, 1985, pp. 363-376.

22. Nelson, R. B.: Simplified Calculation of Eigenvector Derivatives. AIAA Journal, Vol. 14, No.9, September 1976, pp. 1201-1205.

23. Laurenson, R. M.: Modal Analysis of Rotating Flexible Structures. AIAA Journal, Vol. 14, No. 10, October 1976, pp. 1444-1450.

24. Patel, J. S.; and Seltzer, S. M.: NASTRAN: User's Experiences. NASA TMX-2378, September 1971.

25. Peters, D. A.; Ko, T.; Korn, A.; and Rossow, M. P.: Design of Helicopter Rotor Blades for Desired Placement of Natural Frequencies. Presented at 39th Annual Forum of the American Helicopter Society, May 1983.

26. Pritchard, J. I.; Adelman,H. M.; and Haftka, R. T.: Sensitivity Analysis and Optimization of Nodal Point Placement for Vibration Reduction. NASA TM-87763, 1986.

27. Vanderplaats, G. N.: CONMIN - A FORTRAN Program for Constrained Function Minimization - User's Manual. NASA TMX-62282, 1973.